

the invariants

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theta-saurus rex

Evaluate the integral

$$\int_{-\pi}^{\pi} \frac{\theta \sin \theta}{(2024^{\theta^{2023}} + 1)(2024 + 2023 \cos^2 \theta)} d\theta$$

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no strings attached

Consider some set of binary strings S . A string s is S -free if and only if no string from S appears as a continuous substring in s . Characterise those sets S for which there is no infinitely long S -free string.

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alice in the vicinity of numberland

Alice has the numbers

$$1^1, 2^2, 3^3, \dots, 2025^{2025}.$$

Given any permutation (a_i) ($1 \leq i \leq 2025$) of the sequence we consider the product $P = \prod_{i=1}^{2025} (a_i - i^i)$.

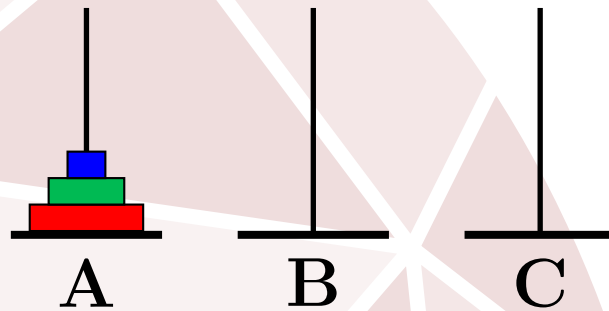
Can P be odd?

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tower of hanoi

In the Tower of Hanoi, there are three pillars – **A**, **B** and **C** – and three discs of decreasing size.



We denote the *game state* by the position of the discs in order by colour; e.g., the initial game state is **A****A****A**.

One can move the top-most disk onto the top of another pillar, provided any blocks below it are wider. How many game states are possible? What's the fewest moves needed to reach every game state?

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stones

I have an $m \times n$ grid ($m \leq n$), and arrange k stones such that each stone shares a row with exactly one other stone, and a column with exactly another stone. Each cell contains no more than one stone.

What's the maximum value of k ? Assuming k is maximised, what's the number of arrangements of the k stones?